HOW TO APPLY THE MULTIPHASE OPTIMIZATION STRATEGY (MOST) IN YOUR INTERVENTION DEVELOPMENT RESEARCH

Module 4
Some conceptual and technical aspects of the factorial experiment

Lesson 2: Interpretation of interaction effects

NYU SCHOOL OF GLOBAL PUBLIC HEALTH
Intervention Optimization Initiative

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In the previous lesson you learned how to:

- Describe the regression model for a factorial experiment
- Explain the difference between dummy coding and effect coding

In this lesson you will learn how to:

- Interpret main effects and interaction effects

Suppose we conducted this experiment and have analyzed the data using a factorial ANOVA, producing a set of predicted outcomes.

Experimental Condition (Cell)	Factorial Experiment				
	MI	*PEER*	*TEXT*	*predicted outcome*	*n*
1	No	No	No	$\hat{Y}_1$	40
2	No	No	Yes	$\hat{Y}_2$	40
3	No	Yes	No	$\hat{Y}_3$	40
4	No	Yes	Yes	$\hat{Y}_4$	40
5	Yes	No	No	$\hat{Y}_5$	40
6	Yes	No	Yes	$\hat{Y}_6$	40
7	Yes	Yes	No	$\hat{Y}_7$	40
8	Yes	Yes	Yes	$\hat{Y}_8$	40

What are we trying to estimate with a factorial experiment?

- Main effect of each factor
  - DEFINITION OF MAIN EFFECT OF FACTOR *A*:
    - Effect of Factor *A* averaged across all levels of all other factors

Let's consider the main effect

- The main effect of *MI* is the effect of *MI* averaged across all the levels of *PEER* and *TEXT*

The main effect of *MI* is the mean of conditions 5—8 MINUS the mean of conditions 1—4.

Experimental Condition (Cell)	Factorial Experiment				
	MI	*PEER*	*TEXT*	*predicted outcome*	*n*
1	No	No	No	$\hat{Y}_1$	40
2	No	No	Yes	$\hat{Y}_2$	40
3	No	Yes	No	$\hat{Y}_3$	40
4	No	Yes	Yes	$\hat{Y}_4$	40
5	Yes	No	No	$\hat{Y}_5$	40
6	Yes	No	Yes	$\hat{Y}_6$	40
7	Yes	Yes	No	$\hat{Y}_7$	40
8	Yes	Yes	Yes	$\hat{Y}_8$	40

Let’s consider the main effect

- The main effect of *MI* is the effect of *MI* averaged across all the levels of *PEER* and *TEXT*

- It is not the effect of *MI* with the other factors set to “no”

If the objective is to estimate the effect of *MI* with all the other factors set to No, an experiment with only conditions 1 and 5 is needed.

Experimental Condition (Cell)	Factorial Experiment				
	MI	*PEER*	*TEXT*	*predicted outcome*	*n*
1	No	No	No	$\hat{Y}_1$	40
2	No	No	Yes	$\hat{Y}_2$	40
3	No	Yes	No	$\hat{Y}_3$	40
4	No	Yes	Yes	$\hat{Y}_4$	40
5	Yes	No	No	$\hat{Y}_5$	40
6	Yes	No	Yes	$\hat{Y}_6$	40
7	Yes	Yes	No	$\hat{Y}_7$	40
8	Yes	Yes	Yes	$\hat{Y}_8$	40

The regression model

$$\hat{Y} = \beta_0 + \underbrace{\beta_1 X_{MI} + \beta_2 X_{PEER} + \beta_3 X_{TEXT}}_{\text{Main effects}} + \underbrace{\beta_4 X_{MI \times PEER} + \beta_5 X_{MI \times TEXT} + \beta_6 X_{PEER \times TEXT} + \beta_7 X_{MI \times PEER \times TEXT}}_{\text{Interactions}}$$

Intercept

The βs correspond to effect estimates.

The regression model

$$\hat{Y} = \beta_0 + \underbrace{\beta_1 X_{MI} + \beta_2 X_{PEER} + \beta_3 X_{TEXT}}_{\text{Main effects}} + \underbrace{\beta_4 X_{MI \times PEER} + \beta_5 X_{MI \times TEXT} + \beta_6 X_{PEER \times TEXT} + \beta_7 X_{MI \times PEER \times TEXT}}_{\text{Interactions}}$$

Intercept

Through tedious but not difficult algebra you can show that

$$\beta_1 = Main\ Effect_{MI}/2$$

What are we trying to estimate with a factorial experiment?

- Interactions
  - DEFINITION OF INTERACTION BETWEEN FACTOR *A* AND FACTOR *B* (assuming each factor has two levels):
    - ½ [(effect of Factor *A* at level 1 of Factor *B*) – (effect of Factor *A* at level 2 of Factor *B*)] averaged across all levels of all other factors

The regression model

$$\hat{Y} = \underbrace{\beta_0}_{\text{Intercept}} + \underbrace{\beta_1 X_{MI} + \beta_2 X_{PEER} + \beta_3 X_{TEXT}}_{\text{Main effects}} + \underbrace{\beta_4 X_{MI\times PEER} + \beta_5 X_{MI\times TEXT} + \beta_6 X_{PEER\times TEXT} + \beta_7 X_{MI\times PEER\times TEXT}}_{\text{Interactions}}$$

Through tedious but not difficult algebra you can show that

$$\beta_4 = Interaction_{MI\times PEER}/2$$

Through tedious but not difficult algebra you can show that <u>for a 2^k experiment</u> (**SEE IMPORTANT CAVEAT****):

$$\beta_4 = Interaction_{MI \times PEER}/2$$

****IMPORTANT CAVEAT:** This equation holds for one definition of the interaction. See sections 3.14 and 4.7 in Collins (2018).

People sometimes worry about interactions because…

1. …they believe they contaminate the main effects, or make the main effects impossible to interpret

Response:

Interactions should always be considered when interpreting main effects

In an effect-coded ANOVA of a balanced factorial experiment, main effects and interactions are uncorrelated

People sometimes worry about interactions because…

1. …they believe they contaminate the main effects, or make the main effects impossible to interpret

Response:

What if you design your experiments so that it's impossible to estimate interactions?

Interactions do not go away just because an investigator chooses not to estimate them (Collins, Dziak, & Li, 2009)

People sometimes worry about interactions because…

2. …they believe there is rarely sufficient power to detect interactions

Response:

Consider a regression weight β_j.

Within a balanced 2^k factorial experiment, statistical power for detection of β_j is <u>identical</u> irrespective of whether β_j corresponds to a main effect or an interaction of any order (Cohen, 1988; Collins, 2018, Ch. 4)

People sometimes worry about interactions because…

2. …they believe there is rarely sufficient power to detect interactions.

Response:

In a balanced 2^k factorial experiment, all else being equal, power is the same for detection of a regression coefficient corresponding to a main effect, a 2-way interaction, a 3-way interaction, etc.

People sometimes worry about interactions because…

2. …they believe there is rarely sufficient power to detect interactions.

Response:

SO if an interaction is defined as the corresponding regression coefficient, in a balanced 2^k factorial experiment, all else being equal, power is identical for detection of main effects and interactions

People sometimes worry about interactions because…

2. …they believe there is rarely sufficient power to detect interactions.

Response:

In this course, we define the interaction as the regression coefficient

HOWEVER, there are different ways to define the interaction, and which one is chosen will affect power estimates

Power associated with detection of interaction effects is a complicated topic. See sections 3.14 and 4.7 in Collins (2018)

One way to think about interactions: They tell the rest of the story of $\widehat{Y}$

Does this tell you what you need to know about the outcome variable?

$$\hat{Y} = \beta_0 + \beta_1 X_{MI} + \beta_2 X_{PEER} + \beta_3 X_{TEXT}$$

Or do you need one or more of the interaction terms?

$$\hat{Y} = \beta_0 + \beta_1 X_{MI} + \beta_2 X_{PEER} + \beta_3 X_{TEXT} + \beta_4 X_{MI \times PEER} + \beta_5 X_{MI \times TEXT} +$$

$$\beta_6 X_{PEER \times TEXT} + \beta_7 X_{MI \times PEER \times TEXT}$$

How to interpret interactions

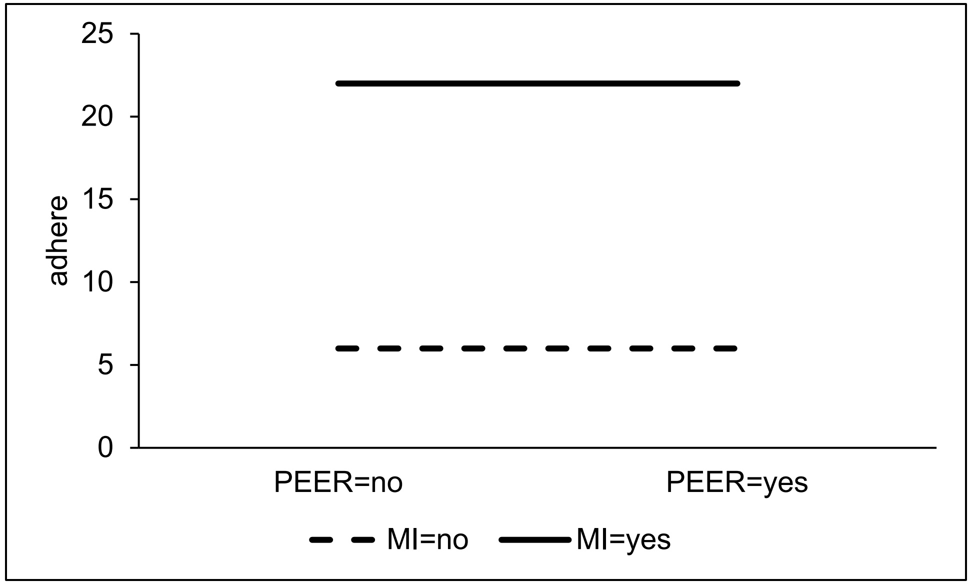
- It is always a good idea to plot an interaction
- You need predicted means (based on $\hat{Y}$s)
  - These will often be marginal means (collapsed across one or more factors)

Experimental Condition (Cell)	Factorial Experiment				
	MI	*PEER*	*TEXT*	*predicted outcome: Adhere*	*n*
1	No	No	No	$\hat{Y}_1$	40
2	No	No	Yes	$\hat{Y}_2$	40
3	No	Yes	No	$\hat{Y}_3$	40
4	No	Yes	Yes	$\hat{Y}_4$	40
5	Yes	No	No	$\hat{Y}_5$	40
6	Yes	No	Yes	$\hat{Y}_6$	40
7	Yes	Yes	No	$\hat{Y}_7$	40
8	Yes	Yes	Yes	$\hat{Y}_8$	40

Let's warm up by looking at plots where there are main effects but no interactions (interactions=zero)

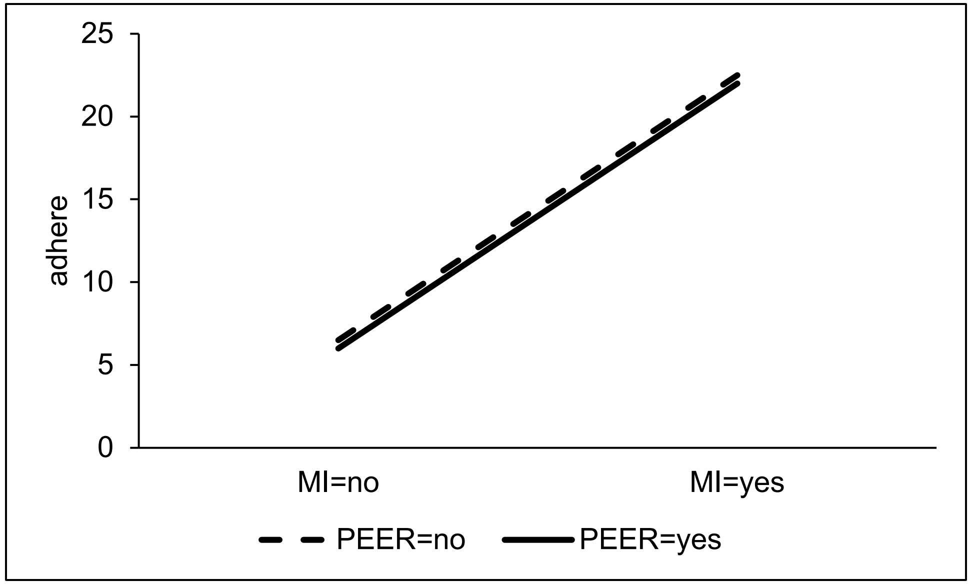
NOTE that in all these plots, we are plotting predicted means

Main effect of *MI*, no main effect of *PEER*, no interaction

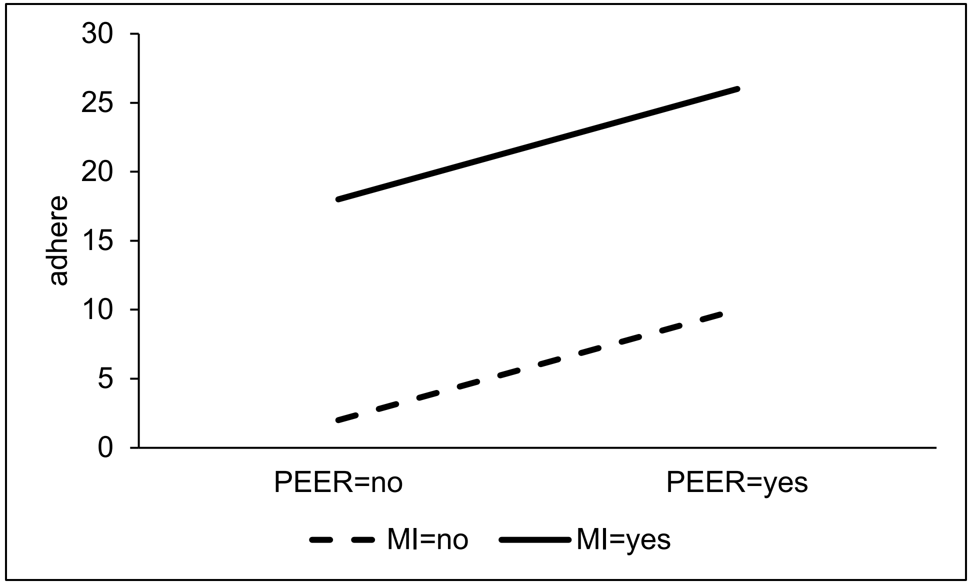


This is a plot of predicted marginal means (in this case, collapsing over *TEXT*)

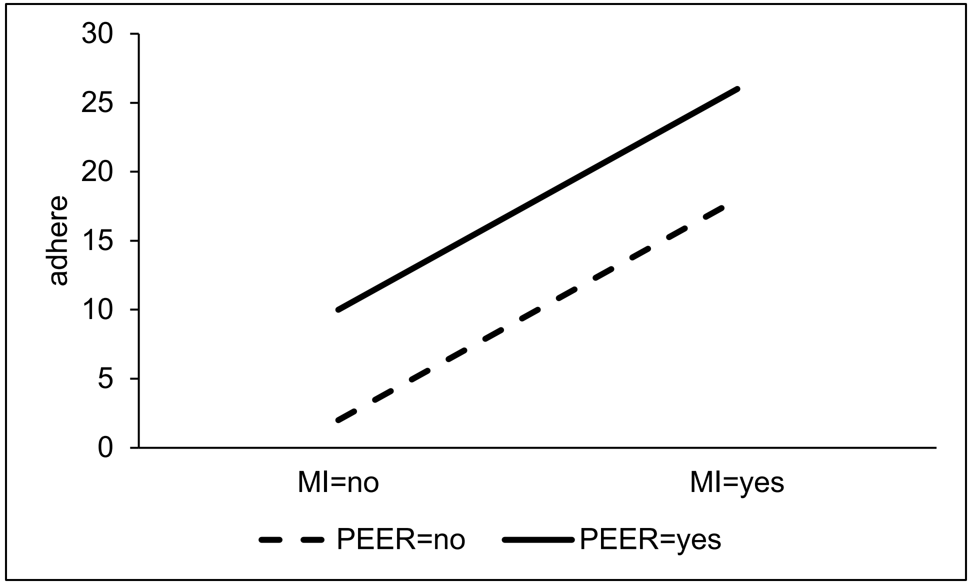
Main effect of *MI*, no main effect of *PEER*, no interaction



Main effects of *MI* and *PEER*, no interaction



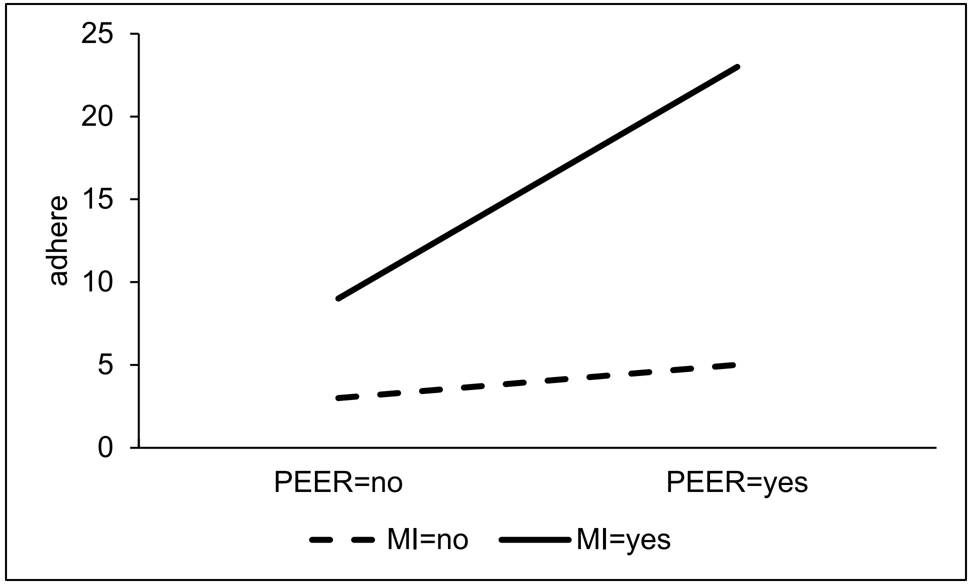
Main effects of *MI* and *PEER*, no interaction



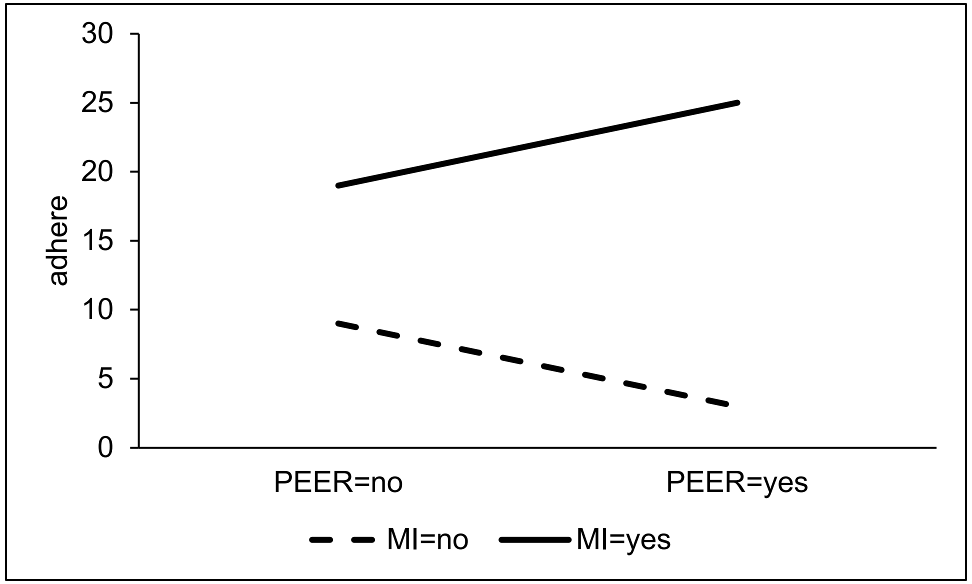
Now we will move to 2-way interactions. Two types of interactions:

- Synergistic
  - Combined effect of two or more factors is <u>more</u> favorable than would be expected based solely on the main effects
- Antagonistic
  - Combined effect of two or more factors is <u>less</u> favorable than would be expected based solely on the main effects

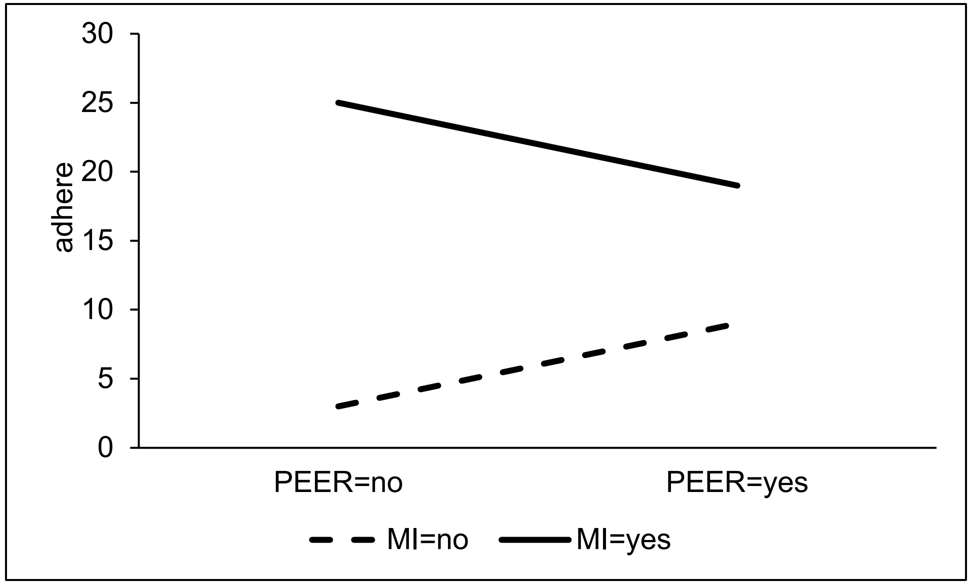
Main effects of *MI* and *PEER*, synergistic *MI*×*PEER* interaction



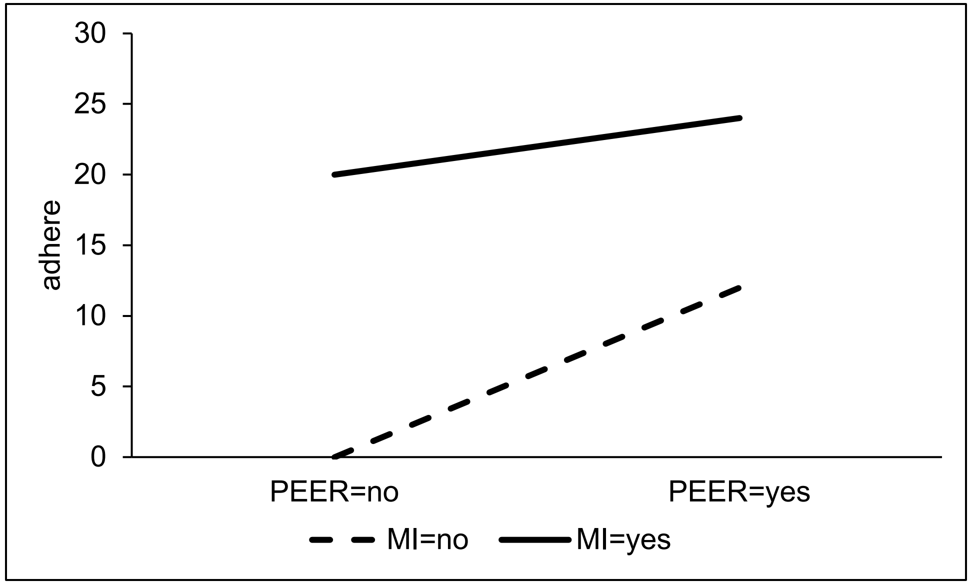
Main effect of *MI*, synergistic *MI×PEER* interaction, no main effect of *PEER*



Main effect of *MI*, antagonistic *MI×PEER* interaction, no main effect of *PEER*

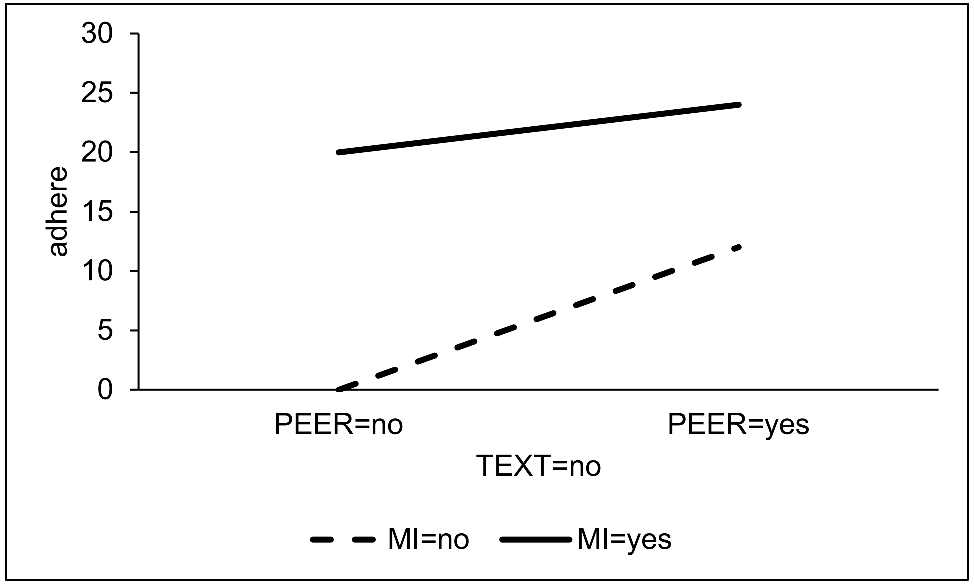


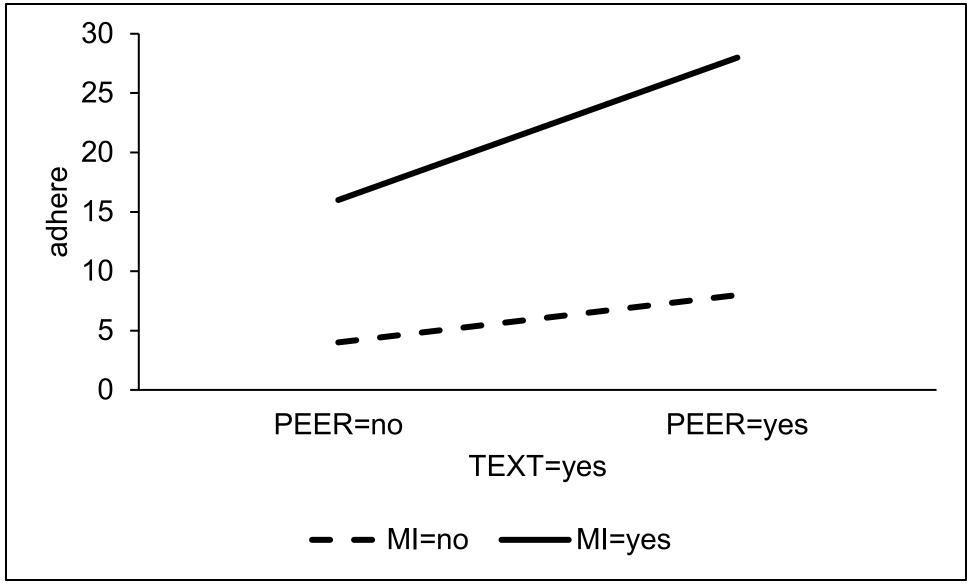
Main effects of *MI* and *PEER*, antagonistic *MI*×*PEER* interaction



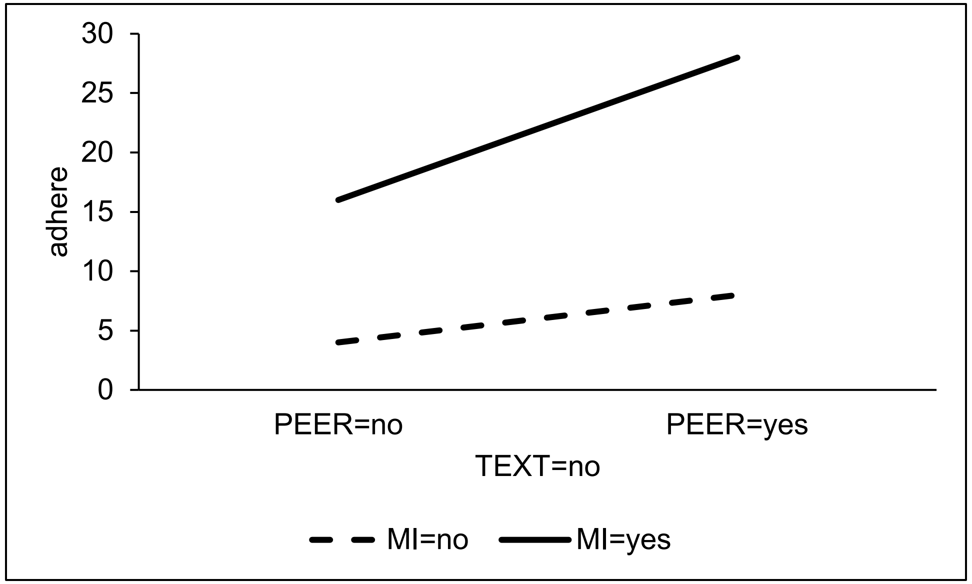
- Now a 3-way interaction—these are more challenging to interpret!

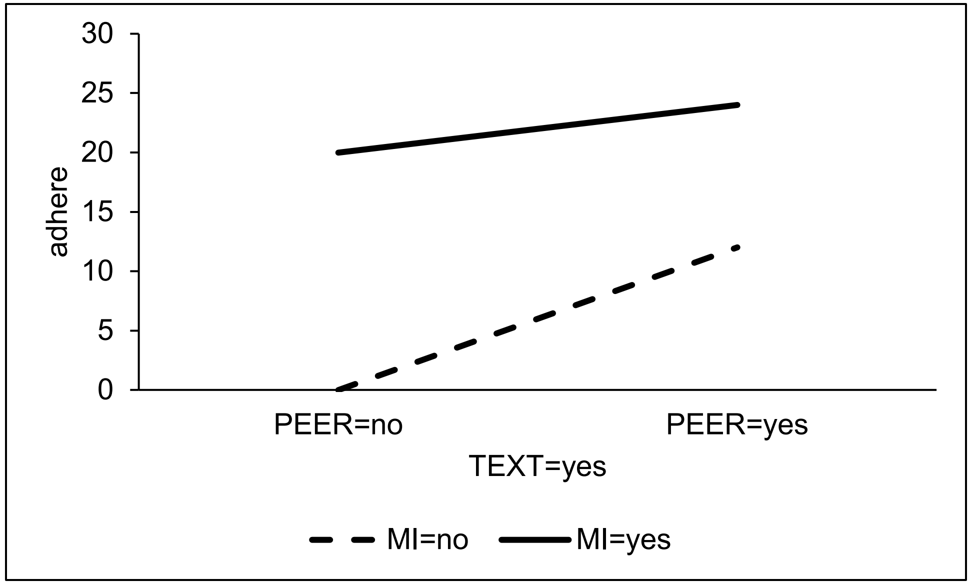
Main effect of *MI*, main effect of *PEER*, synergistic *MI*×*PEER*×*TEXT* interaction (no other effects)





Main effect of *MI*, main effect of *PEER*, antagonistic *MI*×*PEER*×*TEXT* interaction (no other effects)





In this lesson you learned how to:

- Interpret main effects and interaction effects

In the next lesson you will learn how to:

- Understand the basics of powering a factorial experiment

References Cited

- Cohen, J. (1988). Statistical power analysis for the behavioral sciences (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Collins, L. M. (2018). *Optimization of behavioral, biobehavioral, and biomedical interventions: The multiphase optimization strategy (MOST).* New York: Springer.
- Collins, L.M., Dziak, J.J., & Li, R. (2009). Design of experiments with multiple independent variables: A resource management perspective on complete and reduced factorial designs. *Psychological Methods*, *14*, 202-224.